A MODEL OF A POROUS BODY AT HIGH TEMPERATURES

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A one-dimensional porous body is represented by a system of identical axisymmetric channels in a continuous medium. The temperature gradient coincides with the channel axis. To increase thermal resistance, a gas or liquid is passed through the channels. The heat flux is determined. This is possible only by calculating the temperature field, which is necessary, in addition, for drawing conclusions about the behavior of the material. Steady-state heat transfer is considered. The model is such that it is sufficient to examine an elementary cell—an individual channel. General assumptions: local thermodynamic equilibrium, gray-body and diffuse radiation, opaque walls, and isotropic scattering.

1. Equations for temperatures in opening and wall cross sections. The temperature is averaged over each cross section. We introduce the variables

$$x = \frac{l}{D}, \quad \tau = \int_l k\,dl .$$

Here, l is the channel length read from end 1 ($x = 0, \tau = 0$) (Fig. 1); for end 2, $x = x_0$, $\tau = \tau_0$; D is the chosen channel width; $k = \alpha + \beta$ is the attenuation factor; and α and β are the absorption and scattering factors of a ray in the medium. It is convenient to introduce the general variable u, which means x or τ, depending on the form of the functions.

We introduce the functions $W_{uu'}|u' - u|$: the probability that an energy quantum that passes through cross section F(u) will directly strike cross section F(u'). Direct flow includes the quanta that have not interacted with the medium or the walls. In a channel of variable cross section, $W_{uu'}$ is a function of the flow direction. When the fluxes in both cross sections have identical angular distributions, then

$$F(u)\,W_{uu'} = F(u')\,W_{u'u}. \tag{1.1}$$

The first subscript refers to the quantum-source cross section; $\Phi_{u'u}|u' - u|du$ is the probability that an energy quantum that passes through cross section F(u') will directly strike layer du, which is formed by cross sections F(u) and F(u + du), the walls of the channel, and be absorbed or scattered by the medium and reflected from the walls in this layer. Further, we use the distribution

$$\Phi_{u'u}\,|u' - u|\,du = \Phi_{u'F}\,|x' - x|\,dx + \Phi_{u'V}\,|\tau' - \tau|\,d\tau; \tag{1.2}$$

here F and V indicate the lateral surface and volume of layer du; $\varphi_{Fu'}|u' - u|$ is the probability that an energy quantum emitted by the channel walls in layer du will directly strike cross section F(u'); $\varphi_{Vu'}|u' - u|$ is the probability that an energy quantum emitted in the volume of layer du will directly strike cross section F(u'); and $V_{FF'}|x' - x|dx'$ is the probability that an energy quantum emitted by the channel walls in layer du will directly strike the channel walls in layer du'. The probabilities $V_{FV'}|\tau' - \tau|d\tau'$, $V_{VF'}|x' - x|dx'$, and $V_{VV'}|\tau' - \tau|d\tau'$ have similar meanings. According to the phenomenology and definitions, we have

$$\Phi_{u'u}\,|u' - u|\,du = \frac{\partial W_{u'u}\,|u' - u|}{\partial u}\,du \quad (u' > u),$$
$$\Phi_{u'u}\,|u' - u|\,du = -\frac{\partial W_{u'u}\,|u' - u|}{\partial u}\,du \quad (u' < u), \tag{1.3}$$

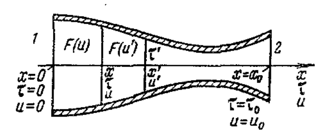

Fig. 1

$$\varphi_{Fu'}\,|u' - u| = \frac{F(u')}{S(x)}\,\Phi_{u'F}\,|x' - x|,$$
$$\varphi_{Vu'}\,|u' - u| = \frac{F(u')}{4F(u)}\,\Phi_{u'V}\,|\tau - \tau'|.$$

Here, $S(x) = dF_b(x)/dx$ is the inside perimeter of the channel multiplied by D; $F_b(x)$ is the inside lateral surface of the channel;
when $u' > u$

$$V_{Fu'}\,|u' - u|\,du' = -\frac{\partial \varphi_{Fu'}\,|u' - u|}{\partial u'}\,du',$$
$$V_{Vu'}\,|u' - u|\,du' = -\frac{\partial \varphi_{Vu'}\,|u' - u|}{\partial u'}\,du',$$

when $u' < u$

$$V_{Fu'}\,|u' - u|\,du' = \frac{\partial \varphi_{Fu'}\,|u' - u|}{\partial u'}\,du'$$
$$V_{Vu'}\,|u' - u|\,du' = \frac{\partial \varphi_{Vu'}\,|u' - u|}{\partial u'}\,du'. \tag{1.5}$$

The distribution of $V_{Fu'}$ and $V_{Vu'}$ over the surface and volume of layer du' has a form similar to that of (1.2):

$$V_{Fu'}\,|u' - u|\,du' = V_{FF'}\,|x' - x|\,dx' + V_{Fv'}\,|\tau' - \tau|\,d\tau',$$
$$V_{Vu'}\,|u' - u|\,du' = V_{VF'}\,|x' - x|\,dx' + V_{VV'}\,|\tau' - \tau|\,d\tau'. \tag{1.6}$$

According to the reciprocal relation

$$V_{F'F} = \frac{s(x)}{s(x')}V_{FF'}, \quad V_{F'V} = \frac{4F(u)}{s(u')}V_{VF'},$$
$$V_{V'F} = \frac{s(u)}{4F(u')}V_{FV'}, \quad V_{V'V} = \frac{F(u)}{F(u')}V_{VV'}. \tag{1.7}$$

In Eqs. (1.3)-(1.7), which relate W, Φ, φ, and V, the principal value is chosen as W. Values of this type are called angular coefficients; they are normalized over the interval [0, 1]. A great deal of study has been devoted to them.

The phenomenological integral equations of energy transfer have the following form:

$$\sigma T^4(\tau) = \frac{g_0(\tau)}{4\alpha(\tau)} + q_{\text{eff}1}\varphi_{V1}(u) + q_{\text{eff}2}\varphi_{V2}(u_0 - u) + \int_0^{x_0} q_{\text{eff}}(x')\,V_{VF'}\,|x' - x|\,dx' + \int_0^{\tau_0} \pi B_{\text{эф}}(\tau')\,V_{VV'}\,|\tau' - \tau|\,d\tau'; \tag{1.8}$$

$$\sigma T_0^4(x) = \frac{q_0}{A} + q_{\text{eff}1}\varphi_{F1}(u) + q_{\text{eff}2}\varphi_{F2}(u_0 - u) + \int_0^{x_0} q_{\text{eff}}(x')\,V_{FF'}\,|x' - x|\,dx' + \int_0^{\tau_0} \pi B_{\text{eff}}(\tau')\,V_{FV'}\,|\tau' - \tau|\,d\tau'; \tag{1.9}$$

$$g_0 = g - \operatorname{div}(c\gamma \mathbf{w} \mathbf{n} T - \lambda_* \mathbf{n}\,\operatorname{grad} T);$$
$$q_0 = q_{\text{res}}\frac{s_0}{s} + \frac{\delta D}{s}\operatorname{div}(\lambda_0\,\mathbf{n}\,\operatorname{grad} T_0); \tag{1.10}$$

$$q_{\text{eff}}(x') = \sigma T_0^4(x') - \frac{1 - A}{A}q_0;$$
$$\pi B_{\text{eff}}(\tau') = \sigma T^4(\tau') - \frac{\beta}{4\alpha k}g_0. \tag{1.11}$$

Here, T and T_0 are the temperatures of the medium and wall, respectively; $\sigma = 5.68 \cdot 10^{-8}$ W/m$^2 \cdot$deg^4; A is the emissivity for the inside surface of the channel; g_0 was taken from [1], where it is called the reduced heat release; g[W/m^3] is the density of chemical heat release in the medium; c, γ, and **w** are the specific heat, density, and velocity of the medium; **n** is a unit vector along the u-axis; λ_* and λ_0 are the thermal conductivity coefficients of the medium and wall; q_{res}[W/m^2] is the density of the resultant flux on the outside surface of the channel, which is positive if the flux enters the wall; for an element (channel) of the porous body $q_{res} = 0$; S_0 is the outside perimeter of the channel multiplied by D; δ[m^2] is the wall cross section. All values in these equations can be functions of the coordinate. The effective flux densities at the ends of the channels q_{eff_1} and q_{eff_2} remain to be explained. Joint solution of the equations for the incident, characteristic, and effective fluxes gives

$$q_{eff_1} = \Big\{ q_{c1} + q_{c2} R_1 W_{12}(u_0) + R_1 \int_0^{x_0} q_{eff}(x)\, \Phi_{1F}(x)\, dx + R_1 \int_0^{\tau_0} \pi B_{eff}(\tau)\, \Phi_{1V}(\tau)\, d\tau + R_1 R_2 W_{12} \Big[\int_0^{x_0} q_{eff}(x)\, \Phi_{2F}(x_0 - x)\, dx + \int_0^{\tau_0} \pi B_{eff}(\tau)\, \Phi_{2V}(\tau_0 - \tau)\, d\tau \Big] \Big\} / [1 - R_1 R_2 W_{12} W_{21}], \quad (1.12)$$

$$q_{eff_2} = \Big\{ q_{c2} + q_{c1} R_2 W_{21}(u_0) + R_2 \int_0^{x_0} q_{eff}(x)\, \Phi_{2F}(x_0 - x)\, dx + R_2 \int_0^{\tau_0} \pi B_{eff}(\tau) \times \Phi_{2V}(\tau_0 - \tau)\, d\tau + R_1 R_2 W_{21} \Big[\int_0^{x_0} q_{eff}(x)\, \Phi_{1F}(x)\, dx + \int_0^{\tau_0} \pi B_{eff}(\tau)\, \Phi_{1V}(\tau)\, d\tau \Big] \Big\} / [1 - R_1 R_2 W_{12} W_{21}] \times$$

$$\times\, q_{c1} = (1 - R_1)\sigma T_1^4, \quad q_{c2} = (1 - R_2)\sigma T_2^4; \quad (1.13)$$

here $q_{eff}(x)$ and $\pi B_{eff}(\tau)$ should be replaced by the right sides of Eqs. (1.11); R_1 and R_2 are the reflection factors (or albedos) of the channel ends; T_1 and T_2 are the temperatures of the ends. Equations (1.8), (1.9), (1.12), and (1.13) make up a system with unknown T, T_0, q_{eff_1}, and q_{eff_2}, from which it is easy to determine any characteristics.

2. Single equation for R = β = 0. In this case, we can, with the most justification, let $T = T_0 = T(u)$. After their multiplication by 4F(u) and S(x), with allowance for (1.4)-(1.6), Eqs. (1.8) and (1.9) take the form

$$4F(u)\sigma T^4 = \frac{F(u)}{\alpha(\tau)} g_0 + g_{*1} F(0)\, \Phi_{1V}(u) + q_{*2} F(u_0)\, \Phi_{2V}(u_0 - u) + \int_0^{u_0} \sigma T^4(u') F(u') \Big| \frac{\partial \Phi_{u'V} |u' - u|}{\partial u'} \Big| du',$$

$$S(x)\sigma T^4 = S(x) q_0 + q_{*1} F(0)\, \Phi_{1F}(u) + q_{*2} F(u_0)\, \Phi_{2F}(u_0 - u) + \int_0^{u_0} \sigma T^4(u') F(u') \Big| \frac{\partial \Phi_{u'F} |u' - u|}{\partial u'} \Big| du',$$

where $q_{*1} = \sigma T_1^4$, $q_{*2} = \sigma T_2^4$.

These equations are multiplied by $d\tau$ and dx, respectively, and then combined. The result is transformed with the aid of (1.2) and divided by du. Then Eq. (1.1) is used. It is useful to introduce the symbols

$$\lambda(u) = \frac{1}{du}\Big[\lambda_*(\tau)\, d\tau + \lambda_0(x) \frac{\delta(x)}{F(u)} dx\Big],$$

$$G(u) = \frac{1}{du}\Big[g \frac{d\tau}{\alpha} + \frac{s_н(x)}{F(u)} q_p(x)\, dx\Big].$$

With

$$4\frac{d\tau}{du} + \frac{S(x)}{F(u)} \frac{dx}{du} = 2\Phi_{uu}(0),$$

which is valid for any "smooth" channel, we obtain the final result

$$2\sigma T^4 \Phi_{uu}(0) = G(u) + \frac{\partial}{\partial u}[\lambda(u)\, \mathbf{n}\, \mathrm{grad}\, T] - \frac{\partial}{\partial u}[c(u)\gamma(u)\mathbf{w}(u)\mathbf{n}T] - q_{*1}\frac{\partial W_{u1}(u)}{\partial u} + q_{*2}\frac{\partial W_{u2}(u_0 - u)}{\partial u} + \int_0^{u_0} \sigma T^4(u') \Big| \frac{\partial^2}{\partial u\, \partial u'} W_{uu'} |u' - u| \Big| du'. \quad (2.1)$$

The value $W_{uu'}|u' - u|$, which is directly calculable, is a function of the angular distribution for the radiation in cross section F(u). Here, however, it figures as the double integral of the function V and is therefore determined by this functions, regardless of the actual angular distribution. According to the conditions of the problem, V is calculated for diffuse radiation of the surfaces and a spherical radiation indicatrix for a volume element. Here, W is the same as for isotropic flux in a cross section, and the use of (1.1) is valid.

The flux equation is obtained from (2.1) by multiplying it by du and integrating in the interval [0, u]

$$\int_0^u G(u)\, du + \lambda(u)\, \mathbf{n}\, \mathrm{grad}\, T \Big|_0^u - c\gamma\mathbf{w}\mathbf{n}T \Big|_0^u + q_{*1}[1 - W_{u1}(u)] - q_{*2}[W_{12}(u_0) - W_{u2}(u_0 - u)] - \int_0^{u_0} \sigma T^4(u)\, dW(u) - \int_0^u \sigma T^4(u') \frac{\partial W(u - u')}{\partial u'} du' + \int_u^{u_0} \sigma T^4(u') \frac{\partial W(u' - u)}{\partial u'} du' = 0. \quad (2.2)$$

In this equation, the total fluxes caused by all types of heat transfer are distinguished

$$q(0) = q_{*1} - q_{*2} W_{12}(u_0) - \int_0^{u_0} \sigma T^4(u)\, dW(u) + c(0)\gamma(0)\mathbf{w}(0)\mathbf{n}T(0) - \lambda(0)\Big(\frac{dT}{dl}\Big)_{l=0},$$

$$q(u) = q_{*1} W_{u1}(u) - q_{*2} W_{u2}(u_0 - u) + \int_0^u \sigma T^4(u') \frac{\partial W_{uu'}(u - u')}{\partial u'} du' - \int_u^{u_0} \sigma T^4(u') \frac{\partial W_{uu'}(u' - u)}{\partial u'} du' + c(u)\gamma(u)\mathbf{w}(u)\mathbf{n}T(u) - \lambda(u)\frac{dT}{dl}.$$

According to the last three equations,

$$q(u) - q(0) = \int_0^u G(u)\, du,$$

or, after differentiation,

$$\frac{dq(u)}{du} = G(u), \quad \frac{dq(u)}{dl} = G(u)\frac{du}{dl} = g_*(u),$$

where g_*[W/m^3] is the specific power of heat release by external and internal sources. For the three-dimensional problem, we have div q = $= g_*$, which this symmetric with respect to the already known relation div $q_{rad} = g_0$, where q_{rad}[W/m^2] is the radiant flux.

Equation (2.2) can be further simplified by eliminating the last derivative, if we let λ = const. For this, it is again multiplied by du and integrated:

$$\int_0^u du \int_0^u G(u)\,du + q(0)\,u =$$

$$= q_{*1}\int_0^u W_{u1}(u)\,du - q_{*2}\int_0^u W_{u2}(u_0 - u)\,du +$$

$$+ \int_0^{u_0} \sigma T^4(u)\,W_{1u}(u)\,du - \int_0^{u_0} \sigma T^4(u')\,W_{uu'}\,|u' - u|\,du'$$

In view of the multiplicity of independent parameters, a general solution of these equations, which is possible in numerical form, is advisable only in a specific practical problem. It will be useful to examine a number of particular solutions.

3. Effect of scattering on temperature field in medium. In (1.11.2), the physical meaning of B_{eff} is explained by the equation $\pi B_{eff} = \sigma T^4_{eff}$, where T_{eff} is the effective temperature, which is equivalent to the temperature of a volume elemnt that emits the same flux but as a characteristic, completely thermal flux. Let the conditions at the boundaries of the channel, the field k, and g_0 be given; only β/k varies. Then we have T_{eff} = const. This theorem is proven logically. By convention, the total heat release is given, and α is the reemission coefficient. The flux distribution and the fluxes themselves are indifferent to origin: reemission or scattering. Rewriting (1.11.2) as

$$\sigma T^4 = \sigma T^4_{eff} + \frac{\beta}{4\alpha k} g_0 \qquad (3.1)$$

gives a function of one variable $T(\beta/k)$. The result is noteworthy: a) when $g_0 = 0$, the temperature is independent of the relationship of α and β; b) when $g_0 \neq 0$, the effect of β/k is determined by the sign of g_0; c) when $\beta/k \to 1$, $T \to \infty$ or $T \to 0$, depending upon the sign of g_0. It is obvious that when $\beta/k = 1$, the conditions are singular. Here, the radiation transfer is autonomous, i.e., independent of heat conduction and convection. The relationship between T and T_{eff} or the radiant temperature disappears.

Our condition of constancy for the field g_0 is possible only when heat conduction or convection is insignificant. In fact, when $\beta/k \to 1$, we have $|g_0| \to 0$. The opposite case, when heat conduction and convection practically completely determine the temperature field, admits of simple analysis. Then in (3.1) we have $T \approx$ const, but $|g_0| \to 0$ when $\beta/k \to 1$. We obtain the function $T_{eff}(\beta/k)$, where the roles of the signs of g_0 are changed. The signs of g_0 are easily established for the beginning and end of the channel. This analysis remains valid for any scattering indicatrix. In a number of cases, its effect is smaller by one order of magnitude than the effect of β/k [2]. For large particles and small optical thicknesses, the indicatrix can be represented as spherical and, in part, maximally extended, so that its effect is taken into account directly.

4. Channel without divergence of total flux. Heat conduction and radiation. Channel ends black. If there are no combustion processes, phase transitions, etc., in the porous body, the divergence of the total energy flux is zero. Combined energy transfer by heat conduction and radiation (convection is absent) when $\beta = 0$ is analyzed below. An approximation of the independent application of fluxes of heat conduction q_c and radiation q_{rad} that is used in practice [3] is examined. It follows from general considerations that as the channel walls "whiten" the radiant flux becomes all the more independent. When R = 1 and $\lambda_* = 0$ (for the medium) it is completely independent; the same occurs when $\beta/k = 1$ and R = 1. The case when $\beta = 0$ and R = 0 is less favorable. Also less favorable is the approximation of a plane layer in a homogeneous medium (degenerate channel), since the interaction of heat conduction and radiation is strongest here. Here we have numerical solutions of the exact equations [4]. The independent heat-conduction flux is calculated by

$$q_c/\sigma T_1^4 = \frac{4N}{\tau_0}(1 - \theta_2)\left(\theta_2 = \frac{T_2}{T_1},\quad N = \frac{\alpha\lambda}{4\sigma T_1^3}\right). \qquad (4.1)$$

The independent radiation flux is

$$q_{rad}/\sigma T_1^4 = D(1 - \theta_2^4), \qquad (4.2)$$

where D is the probability that an energy quantum that strikes a layer will pass through it directly or after reemission.

Comparison of the sum of fluxes from (4.1) and (4.2) with the exact values is demonstrated in [3]. The maximum error is 11%, when q_c and q_{rad} are comparable. The approximate calculation gives an understated result. It was noted above that for channels the method gives a smaller error and is, on the whole, acceptable. According to the definition of the temperature field, there is no method that is equivalent in generality and accuracy.

5. Effect of optical constants on channel ends (continuation of section 4). Specific analysis is also possible for a plane-parallel layer. The number of arguments increases to five: τ_0, θ_2, N, A_1, and A_2; therefore, approximate analytic relations are very desirable. An independent calculation of fluxes q_c and q_{rad} was attempted in [3] under these conditions. Solutions from the exact equation of [5], obtained when $A_1 = A_2 = A$, were used there. The discrepancy, however, is now too great—up to 250%. An approximation formula for small parameters N is given below.

First, we should discuss the radiant-flux formula [6, 7]

$$q_{rad}/\sigma T_1^4 = \frac{1 - \theta_2^4}{r} = \frac{1 - \theta_2^4}{R_1/A_1 + R_2/A_2 + 1/D}, \qquad (5.1)$$

where r is the total resistance to radiant flux. The accuracy of the formula is determined by D. Table 1 gives the most accurate D values for various sources. Evidently, D_1 (according to [8]) when $\tau_0 \le 4$ is somewhat overstated. D_4 (according to [10]) is published for the first time in explicit form. The D values in the last column were obtained by processing all data; they are used in subsequent calculations. A good approximation is given by

$$D = \{1 + 0.75\,\tau_0 + 0.06\,[1 - \exp(-3\,\tau_0)]\}^{-1}.$$

It follows from calculations [8, 11] that q_{rad} is not highly sensitive to the temperature field in the layer. Therefore, the radiant flux can be calculated by (5.1) for any N. To show in pure form the error of the formula proposed below, q_c^0 is taken not from (4.1) but from calculations of the exact equation when $A_1 = A_2 = A$—i.e., from the difference between q and q_{rad}. The main thesis is that when the heat conduction of the medium is low, the temperature and its gradient at the wall are determined by the radiant temperature. At small N, therefore,

$$q_c = q_c{}^0\,\frac{T_1 - T_g}{T_1 - T_b}, \qquad (5.2)$$

where T_b and T_g are the radiative temperatures at wall 1 for black and gray walls. Assuming that T_b and T_g are completely determined by radiation, we use the system of equations

$$p = \frac{T_b^4 - T_2^4}{T_1^4 - T_2^4} = \frac{\sigma T_g^4 - q_{eff2}}{q_{eff1} - q_{eff2}}, \quad q_{eff1} = \sigma T_1^4 - R_1 A_1^{-1} q_{rad},$$

$$q_{eff2} = \sigma T_2^4 + R_2 A_2^{-1} q_{rad}.$$

According to [11, 12],

$$p = \frac{0.5 - E_3(\tau_0) + \tau_0\,[1 - F_2(\tau_0)]}{1 + \tau_0 - \exp(-\tau_0)}.$$

A more exact formula is given in [11]. The value q_{rad} is calculated by (5.1). Finally, the thermal flux is determined by

$$q = q_m{}^0\delta_* + q_{rad}$$

$$\left(\delta_* = \frac{1 - [\theta_2^4 + (1 - \theta_2^4)\,r^{-1}(p/D + R_2/A_2)]^{0.25}}{1 - [\theta_2^4 + (1 - \theta_2^4)\,p]^4}\right). \qquad (5.3)$$

The value δ_* indicates the increase in heat-conduction flux due to the presence of radiation. Now the effect of A_1, A_2, θ_2, and τ_0 is established by elementary analysis, wherein A_1 and A_2 have opposite

Table 1

Comparison of D-Values for a Plane-Parallel Layer

τ_0	D_1 from [6]	D_2 from [9]	D_3 from [7]	D_4 from [10]	D
0.1	9159	9158	9157	—	9158
0.2	8496	—	8491	8492	8492
0.3	7941	—	7934	7936	7936
0.4	7467	—	7458	7459	7459
0.5	7051	7043	7040	7041	7041
0.6	6683	—	6672	6663	6672
0.7	6354	—	—	—	6343
0.8	6057	—	6046	—	6046
0.9	5789	—	—	—	5778
1	5543	5532	5532	—	5532
1.5	—	—	4572	—	4572
2	3905	3896	3900	—	3900
2.5	—	—	3401	—	3401
3	3019	—	3016	—	3016
4	2461	2450	—	—	2458
5	2078	—	—	—	2076
6	1798	1798	—	—	1798
7	1584	—	—	—	1584
8	1416	1416	—	—	1416
9	1280	—	—	—	1280
10	1168	1168	—	—	1168

effects. To check (5.3), we used the results of [4,5], where 0.01 is taken as the minimum value of N. The results are presented in Table 2. When $\tau_0 = 1$, $\theta_2 = 0.5$, and $A = 0.5$, an anomaly is noted. This is evidently explained by inaccuracy of q_c^0. As τ_0 increases, the error increases, due to the decreasing role of radiation. The results are better if the fact that heat conduction reduces radiant flux is taken into account. As is apparent, (5.3) is applicable when $N < 0.01$ and for comparatively thin layers. The error increases appreciably, however, when q_c^0 from (4.1) is used. Unfortunately, in a study of the effect of A_1 and A_2 on heat transfer in channels at small N, a plane-parallel layer (as a degenerate channel) gives the least error, where, according to the theoretical premises adopted, the closest interaction of the material with the radiation must occur. But in a channel, the lateral surfaces intensify heat conduction. Finally, in (5.1), as applied to a channel, the calculation of r should be dealt with separately.

6. Resistance to radiant flux by an axisymmetric channel with piecewise smooth profile and adiabatic walls. In the preceding section, it was shown that the radiant flux can be assumed to be independent. This is equivalent to the condition of wall adiabaticity, under which the resistance to radiant flux is determined most simply. The channel in Fig. 1 should now be considered one of many sections connected in series. The transmission coefficient of the i-th section in the positive direction is D_i. Its meaning is similar to the meaning of D for a layer (see Section 4). In the opposite direction, D_i is calculated from the reciprocal relation. A solution of the problem in the case of molecular flow was published earlier [13], where the general method was compared with a number of others by the example of two sections. By analogy, we obtained the following elementary resistances relative to the input cross section F_1 (input to first section):

a) resistance of emitting end

b) resistance of end 2 (energy sink)

(As is apparent, even when $F_1 = F_2$, the resistances of the ends have asymmetric formulas, which contradicts the definition in [14], where two planes are considered.)

c) resistance of i-th section of channel $(1/D_i - 1)\,F_1/F_i$, where F_i is the cross section of the input to section i;

d) resistance of aperture—input to i-th section

$$\frac{F_1}{F_i}\,\frac{F_{i-1} - F_i}{F_{i-1}},$$

where F_{i-1} is the cross section of the adjacent (i − 1)-th section on the adjacent end. When $F_i = F_{i-1}$ and $F_i > F_{i-1}$, the resistance of the aperture is zero.

On the basis of the continuity of all channel elements, their resistances are added. It is easy to obtain a corollary of the second law of thermodynamics: the reciprocal relation

$$r_+ / F_1 = r_- / F_2,$$

where r_+ and r_- are the channel resistances in the forward and reverse directions. The error of such a general example is determined by the fact that D_i is customarily calculated for diffuse flux entering the section. However, for all sections except the first it is, generally speaking, not diffuse. For two circular sections of equal length and diameter with a transparent medium, the method has been verified [15]. A maximum error of 5.6% was obtained for sections of medium length ($l/D = 2$). Approximate calculation gives a result that is clearly overstated (for resistance).

7. Convection. According to the conditions of the problem, only forced convection is of importance. When $\lambda = 0$ and $A_1 = A_2 = 1$, Eq. (2.1) is solved in an approximation of a plane-parallel layer. The radiation that impinges on the medium from the walls is taken into account indirectly by the field of the specific power of heat release g, where it is assumed that g = const (see [16] on two methods of allowing for boundary conditions). Figure 2 shows the variation of the temperature field due to motion of the medium when wn = w, $\alpha = 0.2\ m^{-1}$, $\tau_0 = 2$, $T_2 = 0$, $T_1 = 350°$ K (here, T_1 is the temperature of the medium entering into the layer), and $g/4\pi\alpha = 46.274\ kW/m^2$. The value $4\pi\alpha c\gamma w/g$ is equal to zero (curve a), $1.508 \cdot 10^{-3}\ deg^{-1}$ (curve b), and $2.765 \cdot 10^{-3}\ deg^{-1}$ (curve c). Under these conditions, even for a gas ($c\gamma$ small) the medium moves with low velocity. However, motion greatly affects the temperature at the beginning of the layer. Under

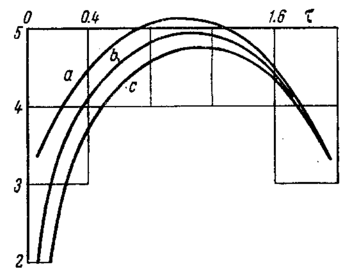

Fig. 2

Table 2

Comparison of Approximate Thermal Fluxes from (5.3) ($q'/\sigma T_1^4$) with Results from Exact Equation [5] ($q/\sigma T_1^4$) for N = 0.01 and $A_1 = A_2 = A$

A	τ_0	θ_2	$q_c^0/\sigma T_1^4$	δ_*	$q_c/\sigma T_1^4$	$q_{rad}/\sigma T_1^4$	$q'/\sigma T_1^4$	$q/\sigma T_1^4$	q'/q
0.5	0.1	0.5	0.215	1.11	0.2385	0.309	0.5475	0.524	1.04
	1	0.5	0.078	1.65	0.1288	0.248	0.3768	0.338	1.11
	1	0.1	0.102	1.54	0.1848	0.274	0.4588	0.390	1.18
	10	0.5	0.012	6.38	0.0765	0.084	0.1805	0.104	1.74
0.1	0.1	0.5	0.215	1.20	0.258	0.049	0.307	0.267	1.15
	1	0.5	0.078	2.23	0.174	0.047	0.221	0.156	1.42
	1	0.1	0.102	2.26	0.230	0.050	0.280	0.222	1.27
	10	0.5	0.012	8.76	0.105	0.035	0.140	0.090	1.56

these conditions, there are no variations at the end of the layer. The conclusions are fully transferable to a channel of constant cross section. An approximate solution is given in [17] for an adiabatic layer with direct allowance for the boundary conditions.

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